A METHODOLOGY FOR PROGNOSTICATING CROP YIELDS WITH SPECIAL REFERENCE TO FERTILIZER USE—REFINEMENTS OF THE MITSCHERLICH APPROACH¹

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1. INTRODUCTION

The yield response functions of fertilizers are extensively used by natural scientists and economists for several purposes, such as (a) recommending 'optimum' dosages of fertilizer applications, (b) estimating the extent to which production can be increased at the farm—, regional—or national levels by using different kinds and quantities of fertilizer, and (c) finding/prescribing the likely nature af resource use pattern and output supply functions, etc. The literature abounds in the estimation of such response functions derived both from experiments and farmers' fields. However, most of these functions are based on the quantities of nutrients actually applied to the soil as fertilizer rather than the quantities of nutrients actually available to the plants. It is well recognized that the latter, in themselves, are function of several variables like, (i) the quantity of various nutrients present in the soil, (ii) other physical and chemical properties of the soil, (iii) the release factor of the fertilizer applied, (iv) cropping systems, (v) climatic and other environmental and physical factors, etc. The magnitude of these variables and their impact differs among locations and over time. Moreover, an empirical production function estimated from data of one experiment often has little relevance beyond the specific year and experimental

^{1.} The equation developed by the author are presented in (4), (5), (6) and (7). The derivation of equations (5), (6) and (7) from (4) has been omitted from this paper due to space constraint and assuming that the readers are familiar with the standard approach of deriving resource requirements from production functions under the usual assumption of the profit-maximizing behaviour of the entrepreneur.

conditions under which it was derived.' Hence there are two major weaknesses in such functions:

- (i) They deal with the response to the applied rather than the available quantities of nutrients, and
- (ii) Their validity is limited to the physical, chemical, environmental and resource (including managerial) situations obtaining on the farms from which they are derived.

2. OBJECTIVE OF THIS PAPER

Conceptually, it is possible to evolve a 'nearly universal' production/response function for a crop if all possible information on conceivable variables affecting crop yields can be obtained and quantified into suitably measurable units. Though a herculean task in itself, it will reduce the need for repeating, under different sets of conditions, the time—, money—, and othe resource consuming experiments, whose results are generally applicable to very limited conditions.

The objective of the exercise presented here is a modest one, i.e. to attempt to formulate a fertilizer response model which will take into account the available quantities of nutrients rather than the applied quantities. It is also a first step towards increasing the validity and applicability of the estimated response functions over a wider mix of other conditions because it takes into account other important variables like the climate, soil productivity, etc. Work is under way on suitable modifications of this approach and to enlarge the model to cover other variables of interest and relevance.

3. Some Basic Concepts and Problems

(i) Mitscherlich response curve³. Mitscherlich, in collaboration with Baule, was one of the first to define the algebraic form of the fertilizer response function as follows:

$$\log A - \log (A - Y) = CX \qquad ...(1)$$
where

A is the maximum possible yield of a crop from the application of input X,

Y is the actual yield obtained from a given quantity of X, and C is a proportionality constant such that 0 < C < 1.

It is a curvilinear response function in that when X, the limiting factor, is added to the soil, the growth of the plant increases

^{3.} Mitscherlich, E.A.: Bodenkunde für Landwirte, Forstwirte and Gartner. Halle. 1947.

but this incremental yield is progressively smaller for each additional unit of X because $C \le 1$. This is seen in (2) below:

$$\frac{dY}{dX} = (A - Y) C \qquad \dots (2)$$

This equation does not allow for a negative $\frac{dY}{dX}$, as $A \geqslant Y$ and 0 < C < 1.

Another objection to Mitscherlich approach is that the values of C, as given by him on the basis of his pot experiments, were assumed to be unaffected by the type of crop, climate or other environmental factors. He calculated the values of C to be. 122, .60 and .40 for N, P_2O_5 and K_2O respectively. The fact is that these values are found to vary under different conditions.

(ii) Baule unit. The quantity of any growth factor which will theoretically produce $Y \approx \frac{A}{2}$ is termed as a Baule unit. As seen in (3) below, $Y \approx A$ when 10 Baule units of any growth factor are applied.

$$Y^{B}=100-(.1\times2(10-X^{B}))$$
 ...(3)

where

 Y^B is the percent of maximum yield i.e. A, and

 X^B is the quantity of a macro-nutrient (N, P_2O_5, K_2O) withdrawn and expressed in Baule units.

Thus at $X^B=1$ (i.e. 1 Baule unit),

$$Y^B = 100 - (.1 \times 2^9) = 48.8\%$$
 or $\approx 50\%$ of A, and at $X^B = 10$, $Y^B = 99.9\%$ of the maximum yield, A.

The values of .122, .60 and .40 correspond to the following quantities (equivalent to one Baule unit) of N, P_2O_5 and K_2O required to produce half of the maximum yield and calculated from (2) and (3) above.

1 Baule unit of N = 2.47 quintals/hectare

1 Baule unit of $P_2O_5 = 0.50$ quintals/hectare

1 Baule unit of $K_2O = 0.75$ quintals/hectare.

Under the approach used by Mitscherlich and Baule, these quantities are fixed.

(iii) Maximum possible yield, i.e. 'A', as defined in (2) above is a conceptual value and difficult to be accurately estimated in practice because there are several factors which may either be or appear to be less significant and unimportant individually (and hence tend to be

omitted from the derivation of response equations), though in aggregate they may be affecting the yields significantly. A more satisfactory and practically useful quantity would be 'maximum yield obtained' (which has been actually measured and hence not a conceptual or hypothetical figure) under commercial (or experimental) conditions. In our model (equations 4, 5, 6 and 7), we use this value of maximum yield (called M) obtained anywhere. The soil, climatic and other conditions prevailing in the location, where prognosis of yield is to be made, are indexed to those under which M was obtained. These indices are calculated by respective subjectmater specialists (e.g. soil scientists for soil) using methods which are highly specialized and complex. Moreover, there are several approaches/procedures of estimating these indices and beyond the scope of this paper. For the interested reader, we are giving below only an elementary treatment of various factors along with some references where detailed discussion of methods can be found.

(iv) Climate. Climate is an important variable affecting crop yields. Different crops/ varieties of crops have, e.g. minimum and maximum tolerable temperatures as well as an optimum range for their growth. The same holds true for the length of the growing period, humidity, etc. These values for different crops are known or can be found out and expressed in the form of a suitable climatic index (R_e in our model), e.g. those developed by Papadakis³ 4) or Stallings⁵). For example, Papadakis first takes factors like evapotranspiration, humidity indices and water surplus. His next step is to determine an index of temperature by considering figures such as (i) mean annual minimum, (ii) average daily maximum of the coldest month, (iii) average of the average daily maxima of the six warmer months, iv) minimum length of the frost-free season. (v) highest monthly average daily minimum, etc. Then the humidity and temerature regimes are determined. Crops are also classified according to their winter resistance, heat requirements, humidity, etc. All these factors are then used to prepare a climate index.

(v) Soil

Different crops perform differently in different soils, depending, e.g. on their (soils') texture, pH, depth, drainage, porosity, etc. The soils can be classified into different measurable productivity classes with the help of chemical, biological and plant tissue tests⁶). The

^{3.} Papadakis, J.: Agri. Geography of the World. Buenos Aires. 1952.

^{4.} Papadakis, J.: Climatic Tables for the World. Buenos Aires. 1961.

Stallings J.L.: Weather Indexes. Journal of Farm Economics, Vol. 42, 1960, pp. 180-186,

response potential of each crop/variety of a crop to various soil properties is thus empirically established and an aggregative index $(R_s$ in our model) prepared. Basic information for preparing such an index (as well as for determing the actual availability of nutrients to the crops, which we discuss in the next paragraph) is available in soil science literature, e.g. Tisdale & Nelson⁶).

(iv) Actual availability of nutrients to the crop:

This is a function of several factors, e.g. (a) quantity of nutrient applied, (b) type of fertilizer used to supply the nutrient, (c) the form of application (basal or top dressing), (d) depth of application, (e) timing of application vis-a-vis the stage of crop growth, (f) quantity present in the soil in available form as determined through chemical and biological analyses, etc. In our model, the end effect of the first five factors (plus some others which may not have been mentioned here) is represented by $\gamma_n N_a$, $\gamma_p P_a$ and $\gamma_k K_a$ where N_a , P_a and K_a are the actual quantities of the three nutrients applied in the form of fertilizer and γ_n , γ_p and γ_k are the release factors showing the proportion of N_a , P_a and K_a made available to the plant because of e.g. (b), (c), (d) and (e) listed above. N_s , P_s and K_s are the quantities of N, P_2O_5 and K_2O that are present in the soil in a form directly available to that crop. Thus the toal quantities of N, P_2O_5 and K_2O actually available to a crop are $\gamma_nN_a+N_s, \gamma_pP_a+P_s$ and $\gamma_k K_a + K_s$ respectively.

(vi) Management:

Management has, for long, been considered to be one of the five essential factors of production. However, the usual response analysis has tended to ignore this factor. Some attempts have been made to prepare a kind of 'management index'. However, such indices have to be prepared with great care to reduce the element of subjectivity because management is not easy to quantify. over, perfect management in crop production is hard to define. Recently, estimates of management efficiency ratings, in terms of percentages, have been developed in California for different crops depending on how difficult (or easy for that matter) they are to manage. Cereals are supposed to be capable of being managed with less difficulty than vegetables (e.g. the management-efficiency ratings of wheat, rice, tomato and anion, are .9, .9, .8 and .8 respectively. However, they are valid only under a given set of circumstances). Some index of individual management factors depending on variables like the experience, education, working environment, etc. can be conceivably constructed.

Tisdale, S.L. and Nelson, W.L.: Soil Fertility and Fertilizers. Macmillan, London. 2nd. ed. 1966.

4. THE MODEL

As we have seen, the pioneering effort of Mitscherlich, in its original form, had limited use for making realistic estimation/prognosis of crop yields. For example, the values of C (and, therefore, quantities of N, P_2O_5 and K_2O making up one Baule unit) were assumed to be constant for each macro-nutrient irrespective of the type of crop/vatriety. Similarly, effects of soil, water, climate, management, quantities of nutrient present in the soil, release factor of fertilizer, etc. were not explicitly incorporated in the equation. Another problem was posed by the concept of "maximum possible yield". The model presented below in (4) over comes these problems.

Let

- Y be the actual yield obtained or expected to be obtained
- M be the maximum yield obtained so far
- R_o , R_s and R_m be the indices for climate, soil & management respectively and expressed as a ratio to M with maximum effect as 1 i.e. $0 < R_c$, R_s , $R_m < 1$. Though, generally speaking, these 3 indices are supposed to range between 0 and 1, conceivably, their values can exceed 1 if the climatic/soil/management conditions on the farm/in the region, for which the prognosis is to be made, are better (for the given crop) than those under which M was obtained.
- N_a , P_a and K_a be the quantities of N, P_2O_5 and K_2O respectively (in kgs/ha) applied as fertilizer
- γ_n , γ_p and γ_k be the efficiency of applied fertilizers with respect to the availability of N, P_2O_5 and K_2O respectively. For example, if only 75% of the total nitrogen applied through a fertilizer is actually available to the crop, then γ_n is .75.
- N_s . P_s and K_s are the quantities (in kgs/ha) of N, P_2O_5 and K_2O respectively that are already present in the soil in a form directly available to the crop even without any fertilizer.
- BUN, BUP and BUK be the quantities of N, P_2O_5 and K_2O (kgs/ha) respectively which are required to produce M/2 of a given crop. These are known from local experienes/experiments.

Then

$$\gamma M. R_{c}. R_{s}. R_{m} \left[1 - (.001) \left\{ 2 \left(10 - \frac{\gamma_{n} N_{a} + N_{c}}{BUN} \right) \right\} \right] \\
\left[1 - (.001) \left\{ 2 \left(10 - \frac{\gamma_{p} P_{a} + P_{s}}{BUP} \right) \right\} \right] \\
\left[1 - (.001) \left\{ 2 \left(10 - \frac{\gamma_{k} K_{a} + K_{s}}{BUK} \right) \right\} \right] ...(4)$$
If $D = \frac{\gamma_{n} N_{a} + N_{s}}{BUN}$

r.e. the quantity (in Baule Units) of nitrogen actually available to the

$$E = \frac{\gamma_p P_a + P_s}{BUP} \text{ and } F = \frac{\gamma_k K_a + K_s}{BUK}$$

are the quantities (expressed in Baule units) of P_2O_5 and K_2O actually available to the cup, then

$$Y=M. R_e. R_s. R_m [1-(.001) \{2^{(10-D)}\}] [1-(.001)\{2^{(10-E)}\}\}$$

[1-(.001) $\{2^{(10-F)}\}\}$... (4a)

The resource demand equations for N, P_2O_5 and K_2O as derived from (4) or (4a) are given in (5), (6) and (7). The optimal quantities of these nutrients to be applied in the form of fertilizer can be obtained by the simultaneous solution of these three equations viz (5), (6) and (7) with three unknowns.

For the sake of convenience, we write

G for
$$[1-(.001) \{2^{(10-D)}\}]$$

H for $[1-(.001) \{2^{(10-E)}\}]$,

and

$$J$$
 for $[1-(.001) \{2^{(10-F)}\}]$,

Let

 N_a^* , P_a^* , K_a^* be the optimal quantities of N, P_2O_5 , and K_2O respectively (in kgs/ha) applied through fertilizer. P_{na} , P_{pa} and P_{ka} be the prices per kg of N_a , P_a and K_a respectively, and P_y be the price per kg of output,

Then

$$BUN \left[10 - \frac{\log \left(\frac{P_{na}}{P_{y}} - \log \left[\left\{ (M.R_{c}.R_{s}.R_{M})I_{n}2(.001) \left(\frac{\gamma_{n}}{BUN} \right) \right\} \{H\} \{J\} \right] \right] - N_{s}}{\log 2} \right]...(5)$$

$$BUP \left[10 - \frac{\log\left(\frac{P_{ng}}{P_{y}}\right) - \log\left[\left\{(M.R_{o}.R_{s}.R_{M})l_{h}2(.001)\left(\frac{\gamma_{x}}{BUP}\right)\right\}\left\{G\right\}\left\{J\right\}\right] \right] - P_{s}}{\log 2} \dots (6)$$

$$BUK \left[10 - \frac{\log\left(\frac{P_{ka}}{P_{y}}\right) - \log\left[\left\{(M.R_{v}.R_{s}.R_{M})l_{h}2(.001)\left(\frac{\gamma_{p}}{BUP}\right)\right\}\left\{G\right\}\left\{H\right\}}{\log 2} \right] - K_{s}$$

$$K_{a} = \frac{\gamma_{k}}{2} \dots (7)$$

5. Illustration

The following hypothetical but simple example will help in explaining the concepts and operation of the model.

In a given region with not very uniform soil and climatic conditions, the highest yield of wheat of a given variety, W_1 , recorded to be 6000 kgs/ha at location L_1 which has highly favourable conditions of soil, climate and management. To get hearly 3000 kgs/ha. (i.e. half of the maximum yield recorded) under similar conditions, 150 kgs/ha of N, 20 kgs/ha of P₂O₅ & 15 kgs/ha of K_2O are needed by the variety. A farmer in another location L_2 of that region has the following conditions: (i) The climate in L_2 with respect to the production of W_1 (in terms of temperature, rainfall, etc) seems to be only 80% as good as that in L_1 . (ii) On the basis of the texture, structure, etc. the suitability of the soil on which the farmer proposes to raise W_1 is also lower. It seems that no more than 75% of the maximum yield could have been obtained on this type of soil even in L_1 . The farm soil analysis (in L_2 where W_1 is proposed to be grown) further shows that the quantities of N, P_2O_5 and K_2O (in kgs/ha) already available to the plants from the soil are 100, 200 and 75 respectively. The efficiency of fertilizers (i.e. the proportion of nutrient taken by the plant from the fertilizer) to be applied by the farmer is 1.0 for N, 0.5 for P_2O_5 and 0.6 for K_2O . Under the assumptions that (i) the yield pattern of W_1 follows the Mitscherlich response behaviour and (ii) the managerial ability of this farmer is as good as of the one on whose field the yield of 6000 kgs. was obtained, the farmer in L_2 is interested in knowing answers to the following two questions:

I. How much yield of W_1 should he expect to get if he applies 200 kgs of N, 40 kgs of P_2O_5 and 50 kgs of K_2O per ha, as fertilizer?

II. Given that there is no significant response of W_1 to the application of P_2O_5 and K_2O as fertilizer under the present circumstances (high P_s and K_s) on his farm, what would be the economically optimal quantity of N that he should apply as fertilizer (and how much yield of W_1 is he likely to get) if the price of N (including its application costs etc) per kg. is 5 times that of 1 kg. of wheat?

According to the information given above

$$M=6000$$
 kgs

 $BUN=150$ kgs (quantity of N required to produce $M/2$)

 $BUP=120$ kgs (quantity of P_2O_5 required to produce $M/2$)

 $BUK=15$ kgs (quantity of K_2O required to produce $M/2$)

 $R_c=.80$
 $R_s=.75$
 $N_s=100$ kgs

 $P_s=200$ kgs

 $K_s=75$ kgs

 $\gamma_n=1.0$
 $\gamma_p=0.5$
 $\gamma_k=0.6$
 $R'_m=1.0$

Solution to Problem I:

$$N_a$$
=200 kgs
 P_a = 40 kgs
 K_a = 50 kgs

$$M.R_{c}.R_{s}.R_{m} = (6000) (.8) (.75) (1) = 3600 \text{ kgs}$$

$$**D_{1} = \frac{\gamma_{n}N_{a} + N_{s}}{BUN} = \frac{(1)(200) + 100}{150} = \frac{300}{150} = 2$$

$$E_{1} = \frac{\gamma_{p}P_{a} + P_{s}}{BUP} = \frac{(.5) (40) + 200}{20} = \frac{220}{20} = 11$$

$$F_{1} = \frac{\gamma_{k}K_{a} + K_{s}}{BUK} = \frac{(.6) (50) + 75}{15} = \frac{105}{15} = 7$$

$$G_{1} = [1 - (.001) \{2^{(10-P_{1})}\}] = [1 - (.001) \{2^{(10-2)}\}]$$

$$= [1 - (.001) (2^{8})] = [1.256] = .744$$

$$H_{1} = [1 - (.001) \{2^{(10-E_{1})}\}] = [1 - (.001) (2^{-1})***] = .9995$$

^{**} The subscripts 1 and 2 refer to problems I and II. The notations with subscripts of 1 are for problem I and with 2 for problem II.

^{***} The power of -1 shows an excessive use of P_2O_5 .

$$J_1=[1-(.001) \{2^{(10-F_1)}\}]=[1-(.001) (2^3)]=.992$$

 $Y_1=(3600) (.744) (.9995) (.992)$
=2655.64 kgs or approx. 2550 kgs.

The farmer should expect an yield of approx. 2550 kgs/ha of W_1 .

Solution to Problem II

Since the farmer does not apply any P_2O_5 and K_2O , $P_a=0$ and $K_a=0$, in this case and,

$$E_2 = \frac{P_s}{BUP} = \frac{200}{20} = 10$$
, and $F_2 = \frac{K_s}{BUK} = 5$

Therefore,

$$H_2 = [1 - (.001)\{(2^{10-E_2})\}] = [1 - \{(.001 (1)\}] = .999$$
, and $J_2 = [1 - (.001)\{(2^{10-F_2})\}] = [1 - \{(.001) (2^5)\}] = .968$

$$M.R_c.R_s.R_m = 3600$$

$$\frac{\gamma_n}{BUN} = \frac{1}{150}$$

$$\log\left(\frac{P_{na}}{P_n}\right) = \log 5 = .699$$

$$M.R_c.R_s.R_m = (ln\ 2)\ (.001)\left(\frac{\gamma_n}{BUN}\right).(H_2)\ (J_2)$$

=(3600) (.6923) (.001) (1/150) (.999) (.968)

$$=2.41/150$$

 $\log (2.41/150) = \log 2.41 - \log 150 = .382 - 2.176 = -1.794$

Substituting the above values in equation (5) which is to be used here since we are interested in optimal quantity of N, we get

$$N_{a}*=150 \left[10 - \left\{ \frac{.699 - (-1.794)}{.301} \right\} \right] - 100$$

$$= \frac{257.64 - 100}{1} = 157.64 \text{ or approx. } 150 - 160 \text{ kgs.}$$

$$D_2 = \frac{\gamma_n \ N_a^* + N_s}{BUN} = \frac{(1) (157.64) + 100}{150}$$
$$= \frac{257.64}{150} = 1.7176, \text{ and}$$

$$G_2 = [1 - (.001) \{(2^{10-D_2})\}] = [1 - (.001) (2^{10-1.7176})]$$

= $[1 - (.001) (2^{8.2824})[=[1 - (.001) (311.35)] = .6886$, and

$$Y_2 = (3600) (.6886) (.999) (.968)$$

=2397.23 or approximately 2400 kgs.

The economically optimal quantity of N for the farmer to apply to W_1 would be around 150–160 kgs/ha and the expected yield of wheat is nearly 2400 kgs/ha.

In this example, the farmer was interested in knowing the optimal dosage of N only. If he wants to apply N, P_2O_5 and K_2O together, their optimal quantities can be easily found out by simultaneous solution of the three equations viz. (5), (6) and (7).

6. ADVANTAGES AND LIMITATIONS OF THE MODEL

- (a) The response equation presented in (4) tries to overcome some of the problem's mentioned earlier. The equation, derived under the usual assumptions underlying this form of production function (i.e. total product curve is asymptotic to M and the marginal product curves to zero axis; a Baule unit is the quantity of any growth factor that will produce $Y \cong \frac{M}{2}$; and Y = M when 10 Baule units of this growth factor are applied), also takes into cognizance the maximum yields acutally obtained, climatic effect, soil properties and fertility, management, the release efficiency of the the fertilizer applied and the amount of nutrients available in the soil.
- (b) The changes in the quantity of application of one nutrient are likely to affect the efficiency and the demand for other nutrients, a fact supported by experience and experiments. The model presented here takes this into account where the demand for any nutrient is also a function of the availability of other nutrients and not determined in isolation, under the convenient assumption of other nutrients being constant. However, as we have seen in our illustration, the model easily takes care of fixed supply of nutrients as well.
- (c) Another advantage of the model is that it is flexible and versatile. Additional variables like the quality and quantity of irrigation water and the timing of its application, quantities of different types of fertilizers applied at different times and in different forms, etc. could be included in the equation without affecting its efficiency. Similarly, the impact of micronutrients on yield can be incorporated.
- (d) The approach does away with the fixed value of Baule unit in terms of N, P_2O_5 and K_2O . Appropriate Baule units for a given crop/variety under the existing conditions can be determined (as was the case in our illustration) and used for estimating and predicting yields.

- (e) The model is somewhat complex but easily amenable to computer programming,
- (f) The equation is unrealistic in that it does not allow for diminishing total returns, but in the initial stages (say upto 3-4 Baule units) behaves fairly realistically.
- (g) The model presented above assumes absence of any interaction between climate, soil, management, etc. However, this assumption can be relaxed by including a measure of inter-relationship 'r' between any two or more factors. For example, r_{cs} could be defined as a measure of relationship between given types of soil and climatic conditions under consideration and could be positive zero or negative. The expression (M. R_c . R_s . R_m .) on right side of (4) could then be replaced by, e g., $M[(R_c$. R_s . R_m .) + $(r_{cs}+r_{cm}+...)$] This is one suggested formulation. Exact formulation will depend on the nature of relationship and the rype of response.
- (h) One objection to this equation could be that since the values of R_o , R_s , R_m are all, generally, less than one, their multiplication (as suggested in the model) may tend to underestimate the response especially when the indices of a large number of variables are included. Trail runs of the model with the presently included three indices have not borne this out. However, one way to obviate problem would be to consider only the most limiting factor (the one with the smallest value of index) *i.e.* to take the minimum of R_c , R_s , and R_m .

The methodololy assumes satisfactory determination of R_c , R_s , R_m , γ_n , γ_p , γ_k , N_s , P_s , K_s and the BUP's for different crops/varieties under the existing conditions and the conditions under which M was obtained. M need not be from the locality for which the yield estimates are to be made. As long as we have the required information on the relevant variables for the conditions under which M was obtained and the existing conditions, the model can be used to give a first approximation of the expected yields and the economically optimal quantities of nutrient application under existing conditions. The possibilities of using the model in regional planning in agriculture are immense.